

ALGEBRA

Lines

Slope of the line through $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept equation of line with slope m and y -intercept b :

$$y = mx + b$$

Point-slope equation of line through $P_1 = (x_1, y_1)$ with slope m :

$$y - y_1 = m(x - x_1)$$

Point-point equation of line through $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$$y - y_1 = m(x - x_1) \quad \text{where } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines of slope m_1 and m_2 are parallel if and only if $m_1 = m_2$.

Lines of slope m_1 and m_2 are perpendicular if and only if $m_1 = -\frac{1}{m_2}$.

Circles

Equation of the circle with center (a, b) and radius r :

$$(x - a)^2 + (y - b)^2 = r^2$$

Distance and Midpoint Formulas

Distance between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of $\overline{P_1 P_2}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Laws of Exponents

$$x^m x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn}$$

$$x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{1/n} = \sqrt[n]{x}$$

$$\sqrt[n]{xy} = \sqrt[n]{x} \sqrt[n]{y}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

$$x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

Special Factorizations

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Binomial Theorem

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + nx y^{n-1} + y^n$$

$$\text{where } \binom{n}{k} = \frac{n(n-1) \cdots (n-k+1)}{1 \cdot 2 \cdot 3 \cdots k}$$

Quadratic Formula

$$\text{If } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities and Absolute Value

If $a < b$ and $b < c$, then $a < c$.

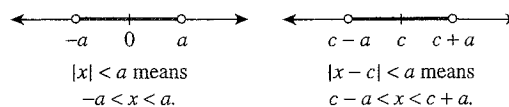
If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

$$|x| = x \quad \text{if } x \geq 0$$

$$|x| = -x \quad \text{if } x \leq 0$$



GEOMETRY

Formulas for area A , circumference C , and volume V

Triangle

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}ab \sin \theta$$

Circle

$$A = \pi r^2$$

$$C = 2\pi r$$

Sector of Circle

$$A = \frac{1}{2}r^2\theta$$

$$s = r\theta$$

(θ in radians)

Sphere

$$V = \frac{4}{3}\pi r^3$$

$$A = 4\pi r^2$$

Cylinder

$$V = \pi r^2 h$$

Cone

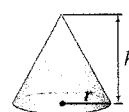
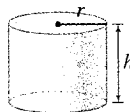
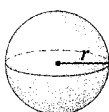
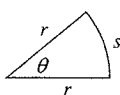
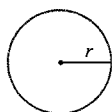
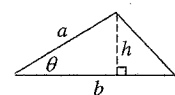
$$V = \frac{1}{3}\pi r^2 h$$

$$A = \pi r \sqrt{r^2 + h^2}$$

Cone with arbitrary base

$$V = \frac{1}{3}Ah$$

where A is the area of the base



Pythagorean Theorem: For a right triangle with hypotenuse of length c and legs of lengths a and b , $c^2 = a^2 + b^2$.

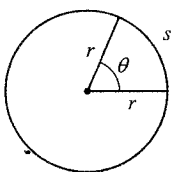
TRIGONOMETRY

Angle Measurement

$$\pi \text{ radians} = 180^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad} \quad 1 \text{ rad} = \frac{180^\circ}{\pi}$$

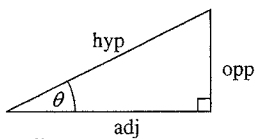
$$s = r\theta \quad (\theta \text{ in radians})$$



Right Triangle Definitions

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$



$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{opp}}{\text{adj}}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{\text{adj}}{\text{opp}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}}$$

Trigonometric Functions

$$\sin \theta = \frac{y}{r}$$

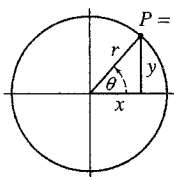
$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}$$

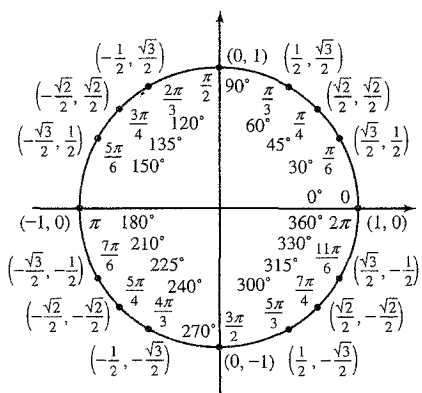
$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$



$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta} = 0$$



Fundamental Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin(-\theta) = -\sin \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cos(-\theta) = \cos \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\sin(\theta + 2\pi) = \sin \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

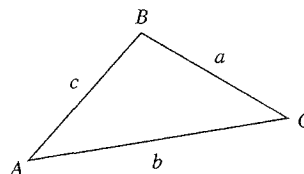
$$\cos(\theta + 2\pi) = \cos \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

$$\tan(\theta + \pi) = \tan \theta$$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Addition and Subtraction Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

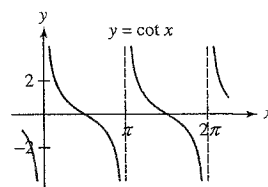
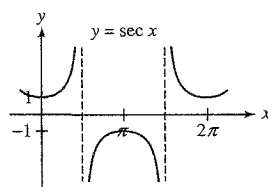
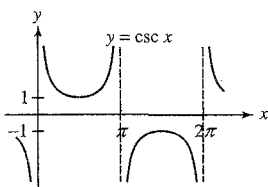
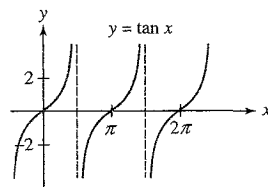
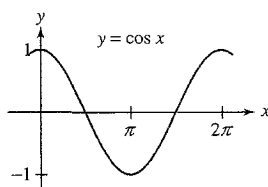
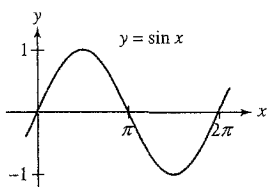
$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

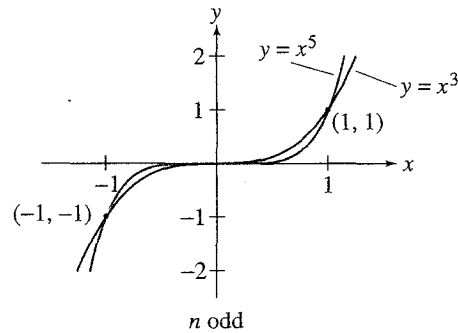
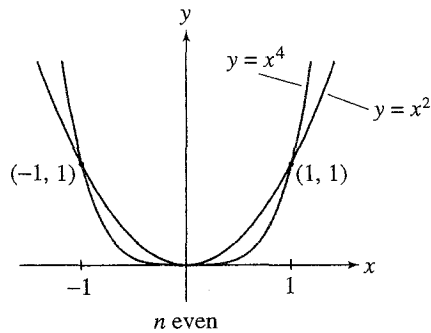
Graphs of Trigonometric Functions



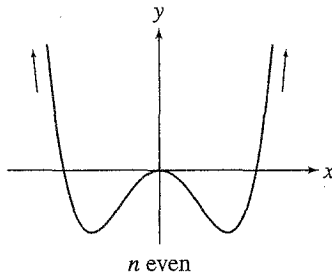
ELEMENTARY FUNCTIONS

Power Functions $f(x) = x^a$

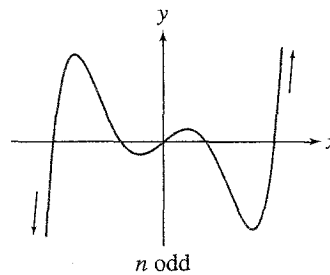
$f(x) = x^n$, n a positive integer



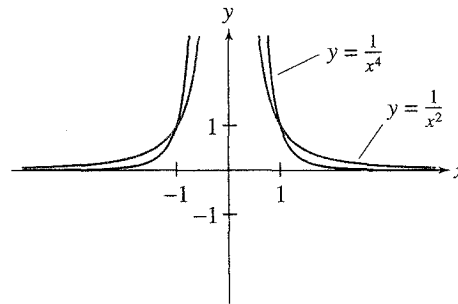
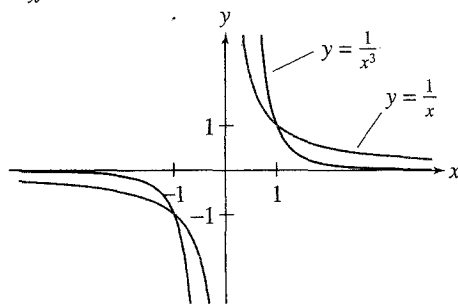
Asymptotic behavior of an even polynomial function



Asymptotic behavior of an odd polynomial function



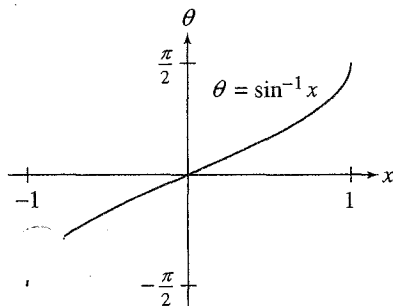
$f(x) = x^{-n} = \frac{1}{x^n}$



Inverse Trigonometric Functions

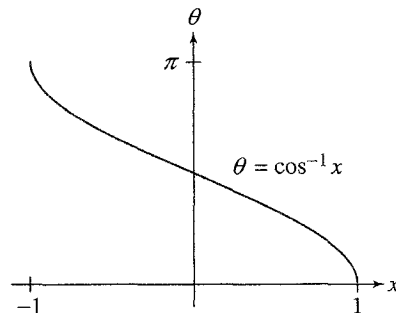
$\arcsin x = \sin^{-1} x = \theta$

$$\Leftrightarrow \sin \theta = x, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$



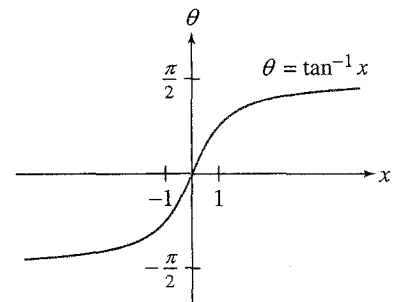
$\arccos x = \cos^{-1} x = \theta$

$$\Leftrightarrow \cos \theta = x, \quad 0 \leq \theta \leq \pi$$



$\arctan x = \tan^{-1} x = \theta$

$$\Leftrightarrow \tan \theta = x, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$



Exponential and Logarithmic Functions

$$\log_a x = y \Leftrightarrow a^y = x$$

$$\ln x = y \Leftrightarrow e^y = x$$

$$\log_a(a^x) = x \quad a^{\log_a x} = x$$

$$\ln(e^x) = x \quad e^{\ln x} = x$$

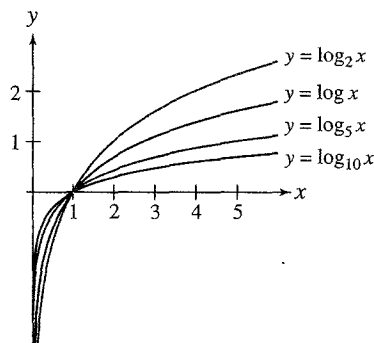
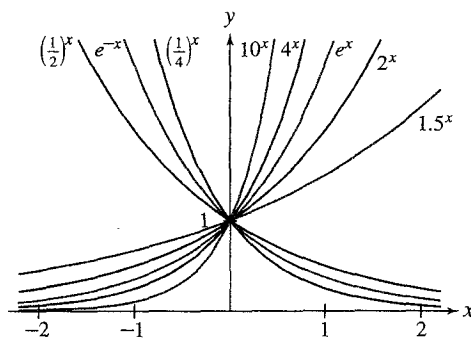
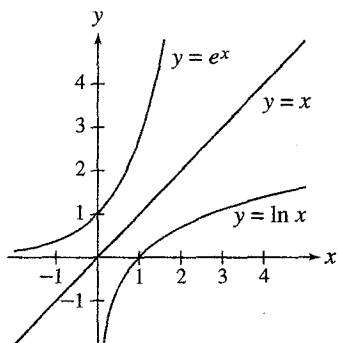
$$\log_a 1 = 0 \quad \log_a a = 1$$

$$\ln 1 = 0 \quad \ln e = 1$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^r) = r \log_a x$$



$$\lim_{x \rightarrow \infty} a^x = \infty, \quad a > 1$$

$$\lim_{x \rightarrow -\infty} a^x = 0, \quad a > 1$$

$$\lim_{x \rightarrow \infty} a^x = 0, \quad 0 < a < 1$$

$$\lim_{x \rightarrow -\infty} a^x = \infty, \quad 0 < a < 1$$

$$\lim_{x \rightarrow 0^+} \log_a x = -\infty$$

$$\lim_{x \rightarrow \infty} \log_a x = \infty$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

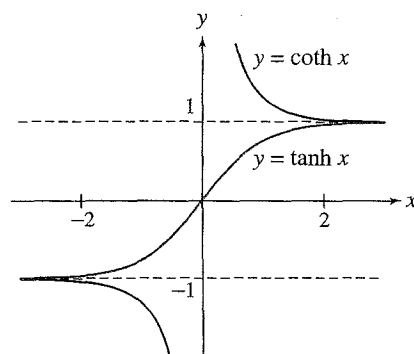
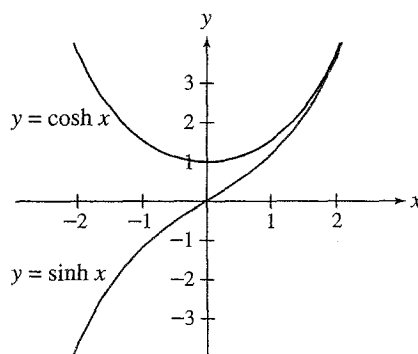
$$\operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$



$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

Inverse Hyperbolic Functions

$$y = \sinh^{-1} x \Leftrightarrow \sinh y = x$$

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

$$y = \cosh^{-1} x \Leftrightarrow \cosh y = x \text{ and } y \geq 0$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}) \quad x > 1$$

$$y = \tanh^{-1} x \Leftrightarrow \tanh y = x$$

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad -1 < x < 1$$

